From which we have

$$T = T_1 - (T_1 - T_2)\exp(-FP). \tag{13}$$

Equation (13), with allowance for the pore cooling criterion [4] and the dimensionless temperature $T^* = (T - T_1)/T_1$, is reduced to the form

$$T^* = T_2^* \exp\left(-K_n \frac{P^*}{\chi} \frac{V_0}{\rho\xi}\right).$$

Figure 3 shows the temperature distribution in the porous half-space in the flow of air, helium, and hydrogen. It can be seen from the graph that, to maintain a certain thermal state in the porous body during cooling with a gas, it is best to use a substance with a lower molecular weight.

NOTATION

T, temperature; c, specific heat of the gas; ρ, density of the gas; λ, effective thermal conductivity of the porous-body–coolant system; τ, β, Chaplygin variables; $n + 1$, degree of filtration (filtration is linear at $n = 0$); $\chi = V_0^n M/P_0\alpha$, dimensionless filtration parameter; $\psi^* = \psi/M$, dimensionless stream function; $P^* = P/P_0$, dimensionless pressure; α, constant characterizing the porous medium and coolant; $\mu(T)$, absolute viscosity of the gas; f(v), function determining the filtration law in each specific case; $x^* = x/d$, dimensionless coordinate; d, characteristic dimension; R, gas constant.

LITERATURE CITED

1. B. M. Smol'skii, P. A. Novikov, and V. A. Eremenko, "Nonsteady problem of pore cooling in the flow of a gas into a vacuum," in: Phase and Chemical Transformations in the Interaction of Solids with a Gas Flow [in Russian], Minsk (1975), pp. 162-167.
2. V. I. Voronin and V. V. Faleev, "Exponential filtration through a porous half-space caused by a source," Inzh.-Fiz. Zh., 33, No. 2, 370-371 (1972).
3. V. I. Voronin and A. N. Glushakov, "Stationary temperature field in pore cooling," Inzh.-Fiz. Zh., 13, No. 6, 921-926 (1967).
4. A. V. Lykov, Heat and Mass Transfer (Handbook) [in Russian], Énergiya, Moscow (1972).

THEORY OF THE FLOW AND CONDUCTION OF INHOMOGENEOUS MEDIA

I. BASIC MODEL OF AN INHOMOGENEOUS MEDIUM

G. N. Dul'nev and V. V. Novikov

UDC 536.21

A basic model of an inhomogeneous medium is outlined and, by a combination of the methods of flow theory and reduction to an elementary cell, an analytic dependence is obtained for the conduction of such a medium.

In studying the conduction of inhomogeneous materials with a random distribution of components, there has been steadily increasing use, in recent years, of a new method of investigation, called flow theory [1-3]. For a binary inhomogeneous system, in which the conductivity of one component $\Lambda_1 \neq 0$ is nonzero, while the other is zero $\Lambda_2 = 0$, the effective conductivity Λ, according to flow theory, is [3]

Leningrad Institute of Precision Mechanics and Optics.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 45, No. 3, pp. 443-451, September, 1983. Original article submitted August 3, 1982.

0022-0841/83/4503-1030$07.50

$$\Lambda = \Lambda_1 \left(\frac{m_1 - m_c}{1 - m_c}\right)^k, \quad m_c = 0.15 \pm 0.03; \; k = 1.6 \pm 0.4. \tag{1}$$

With an accuracy that is acceptable for practical calculations, Eq. (1) describes the effective conductivity of such a binary system and also takes account of the presence of jump conductivity when the volume concentration m_1 reaches some critical value $m_1 = m_c$, the so-called flow threshold.

For materials with a nonzero conductivity ratio of the components, $\nu = \Lambda_2/\Lambda_1$, a combination of the flow and effective-medium methods [1, 2] or of the flow theory and reduction to an elementary cell [3] is used in describing transfer processes. The latter combination is very clear, and leads to good agreement of the calculational and experimental data when $0 \leq \nu \leq 10^{-1}$, while the results of calculation are too high when $\nu > 10^{-1}$. The reason for this disagreement is associated with certain assumptions adopted in analyzing the transfer process and in constructing a model of the inhomogeneous binary system; a refined (basic) model of such a system is considered below.

The structure of a binary inhomogeneous random system is considered and, following the method adopted in [3], elementary cells are modeled for various values of the component concentrations. Note that the particles in the system are isomeric and the whole volume of the material is filled without vacancies. With change in bulk concentration m_1 of the first component in the range $0 \leq m_1 \leq m_c$, the continuous binder includes isolated inclusions (isolated clusters, IC) of the first component (Fig. 1a). The elementary-cell model is a cube of side L, with individual cubic inclusions (IC model) of dimensions l_2 and volume concentration $m_1 = (l_2/L)^3$; the inclusions are a distance $2(L - l_2)$ apart (Fig. 2a).

In the concentration range $m_c \leq m_1 \leq 0.5$, connections appear between the isolated clusters (IC), and the IC are transformed to an infinite cluster (InC); transition occurs abruptly at $m_1 = m_c$ (Fig. 1b). In the model, these structural changes are shown in Fig. 2b, where the isolated clusters are connected by bridges of cross-sectional area l_1^2. When $m_1 = m_2 = 0.5$, two equivalent infinite clusters are formed in the material (Fig. 1c); the corresponding model is shown in Fig. 2c, in the form of an elementary cell of the system with interpenetrating components.

Further increase in the volume concentration of the first component $m_1 > 0.5$ leads to structural change in the opposite order, i.e., the infinite cluster consisting of the second component begins to diminish (Fig. 1d) and, at $m_2 = m_c$, there is an abrupt transition from InC to IC (Fig. 1e), while the infinite cluster of the first component grows continuously. Elementary cells of the model of this structure are shown in Fig. 2e,f.

As shown in [3], the greatest difficulty is in determining the geometric parameters of the elementary cell l_1, l_2 and their relation to the real mean extent L_{InC} of an infinite cluster and its mean cross-sectional area S_{InC}.

The resistance of a binary system consisting of conducting and nonconducting particles ($\nu = 0$) is considered. The resistance R of such a cube is the resistance R_{InC} of the conducting infinite cluster, since the flux propagates only through the conducting component

$$R = R_{InC}, \; R = L/(\Lambda S), \quad R_{InC} = L_{InC}/(\Lambda_1 S_{InC}), \quad S = L^2. \tag{2}$$

It follows from these dependences that

$$\Lambda = \Lambda_1 \bar{S}_1, \quad \bar{S}_1 = S_{InC} L/(S L_{InC}). \tag{3}$$

Comparison of Eqs. (1) and (3) gives

$$\bar{S}_1 = \left(\frac{m_1 - m_c}{1 - m_c}\right)^{1.6}, \tag{4}$$

the law of variation of the relative effective cross section $\bar{S}_1$ of the conducting InC, in which both the complex structure of the InC and the probabilistic character of its formation are taken into account. It is now necessary to relate $\bar{S}_1$ to the geometric parameters of the elementary cell.

The elementary-cell resistance of the model with $\nu = 0$ and "adiabatic" division of the cell is [4]

$$R = L/(\Lambda_1 S_1' H_1), \tag{5}$$

where S_1' is the cross-sectional area of the elementary cell perpendicular to the heat-carrier flux and occupied by the conducting component; H_1 is a correcting function, the meaning of which will be elucidated below.

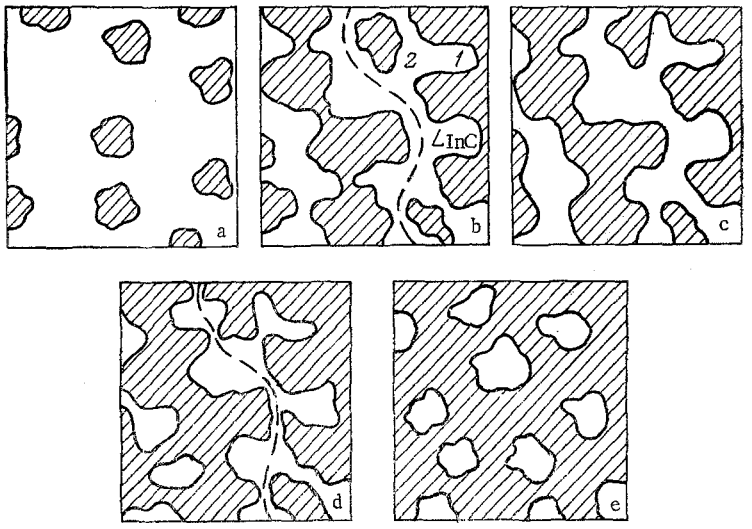

Fig. 1. Plane representation of the structure of a binary inhomogeneous random system: a) $m_1 < m_c$; b) $m_c < m_1 < 0.5$; c) $m_1 = m_2$; d) $0.5 < m_1 < (1 - m_c)$; e) $(1 - m_c) < m_1 < 1$.

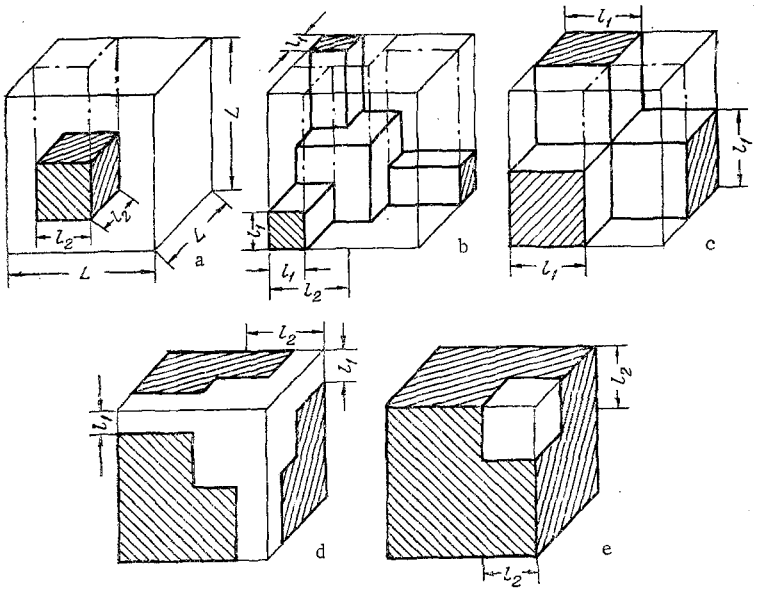

Fig. 2. Model of the elementary cell of a binary inhomogeneous system: a) $m_1 < m_c$; b) $m_c < m_1 < 0.5$; c) $m_1 < m_2$; d) $0.5 < m_1 < (1 - m_c)$; e) $(1 - m_c) < m_1 < 1$.

From Eqs. (2) and (5), the effective conductivity of the elementary cell may be written in the form

$$\Lambda = \Lambda_1 \overline{S}_1' H_1, \quad \overline{S}_1' = S_1'/S. \tag{6}$$

Since the effective conductivities of the sample and the elementary cell are equal, it follows from Eqs. (6) and (3) that $\overline{S}_1' H_1 = \overline{S}_1$ and, in view of Eq. (4), it is found that

$$H_1 = \left(\frac{m_1 - m_c}{1 - m_c}\right)^{1.6} \frac{1}{\overline{S}_1'}. \tag{7}$$

Note that the derivation of Eq. (7) employs Eq. (1), which is in good agreement with the experimental data on the conductivity of an extremely inhomogeneous binary mixture ($\nu = 0$). Hence, the correcting function H_1 allows certain of the constraints imposed above in consutrcting the model to be lifted: specifically, the presence of branches that are closed to the flow of form 1 or individual isolated clusters 2 at $m_1 \geq m_c$ (Fig. 1b). In addition, the errors due to the approximate mathematical description used subsequently in the model are reduced.

An expression for $\overline{S}_1'$ is now determined. The range of concentration variation $m_c \leq m_1 \leq 0.5$ is considered first. In the model of Fig. 2b, the IC are connected by bridges of square cross section, i.e., $S_1' = l_1^2$.

The volume concentration of an infinite cluster with variation $m_c \leq m_1 \leq 0.5$ is related to the model dimensions l_1, l_2, L as follows

$$m_1 = \frac{V_1}{V} = \frac{3l_1^2(L - l_2) + l_2^3}{L^3}. \tag{8}$$

An expression for l_1 of the connecting bridge is obtained from this equation

$$\overline{l}_1^2 = \left(\frac{l_1}{L}\right)^2 = \overline{S}_1' = \frac{1}{3}\,\frac{m_1 - \overline{l}_2^3}{1 - \overline{l}_2}\,; \quad \overline{l}_2 = \frac{l_2}{L}. \tag{9}$$

From Eqs. (4) and (9), $\overline{l}_2$ of the central core of the infinite cluster may be determined: $\overline{l}_2 = f(m_1, m_c)$. However, let us proceed differently: assume that, in the range of concentration variation $m_c \leq m_1 \leq 0.5$, the dimension $\overline{l}_2$ of the central core remains unchanged and equal to that of the IC before jump conduction, that is

$$\overline{l}_2 = (\overline{l}_2)_{\max} = \sqrt[3]{m_c}. \tag{10}$$

Thus, it is found from Eqs. (9) and (10)

$$\overline{S}_1' = \frac{1}{3}\,\frac{m_1 - m_c}{1 - m_c^{1/3}}. \tag{11}$$

If $0.5 \leq m_1 \leq (1 - m_c)$, the components are spatially changed by bridges (Fig. 2d) in comparison with the preceding case $m_c \leq m_1 \leq 0.5$, and S_1' will be

$$S_1' = S - l_2^2 - 2(1 - l_2)\,l_1. \tag{12}$$

Here l_2 is the dimension of the central cube, now consisting of the poorly conducting component ($\Lambda_2 = 0$) and is equal, as before, to $\overline{l}_2 = m_c^{1/3}$, while l_1^2 is the cross-sectional area of the isolational bridges connecting the central cubes. Writing an equation for m_2 analogous to Eq. (8), l_1^2 is determined

$$\overline{l}_1^2 = \frac{1}{3}\,\frac{m_2 - m_c}{1 - m_c^{1/3}}. \tag{13}$$

Then Eq. (12) may be rewritten in the form

$$\overline{S}_1' = 1 - m_c^{2/3} - \frac{2}{\sqrt{3}}\,[(m_2 - m_c)(1 - m_c^{1/3})^{-1}]^{1/2}. \tag{14}$$

If $1 - m_c \leq m_1 \leq 1$, then there are isolated cubes of poorly conducting component in the conducting mass (Fig. 2e); therefore, $S_1' = S - \overline{l}_2^2$; taking into account that $\overline{l}_2 = m_2^{1/3}$, it is found that

$$\overline{S}_1' = 1 - m_2^{2/3}. \tag{15}$$

As already noted, formulas for the function H_1 have been obtained for the case of a limiting conductivity ratio of the components, $\nu = 0$.

The correcting function $H(\nu)$ is now considered for the more general case $\nu \neq 0$, and it is taken into account that it must take the value H_1 at $\nu = 0$ and is equal to unity at $\nu = 1$, since we are dealing here with a single material in which there are no deviations of the current lines, that is

$$H(\nu)|_{\nu=0} = H_1, \quad H(\nu)|_{\nu=1} = 1. \tag{16}$$

This equation is satisfied by the function

$$H(\nu) = H_1 + g(\nu)(1 - H_1), \tag{17}$$

where $g(\nu)$ is an unknown function depending only on ν.

The function $g(\nu)$ is written in the form of a series expansion

$$g(\nu) = \sum_{n=0}^{\infty} a_n \nu^n. \tag{18}$$

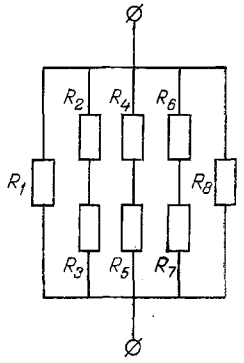

Fig. 3

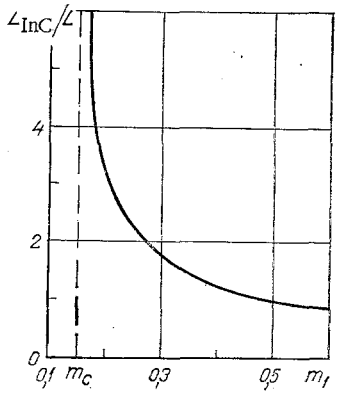

Fig. 4

Fig. 3. Equivalent coupling scheme for the thermal resistances of the elementary cell.

Fig. 4. Length ratio of the infinite cluster in an inhomogeneous medium and the model, with various concentration values of the components.

In determining $g(\nu)$, the starting point adopted is an additional requirement of continuity of the first derivative $d\Lambda/dm_1$ at three points with $m_1 = m_c$, 0.5, and $(1 - m_c)$, where qualitative change in the model occurs: transition from the IC model to the InC model (to a model with equivalent interpenetrating components) and from the InC model to the IC model. The calculations show that this requirement may be satisfied if it is assumed that

$$g(0,\ 2) = 0.8, \quad g(0,\ 5) = 0.9, \quad g(0,\ 8) = 0.98. \tag{19}$$

In addition, the condition in Eq. (16) leads to the result

$$g(0) = 0, \quad g(1) = 1. \tag{20}$$

Thus, Eq. (19) may be limited to the first five terms of the expansion, and the coefficients a_n may be determined from the conditions in Eqs. (19) and (20); after transformation, it is found that

$$g(v) = 5.53v - 8.30v^2 + 3.23v^3 + 0.54v^4. \tag{21}$$

The transfer of heat carriers in the given model, with $0 \leq \nu \leq 1$, is described by dividing it into infinitely thin planes that are impenetrable to the flow [3, 4]. The total mean flux j from the leading to the trailing wall of the cube (Fig. 2b) is divided into a series of mean fluxes

$$j = j_1 + j_{12} + 2j'_{12} + j_2. \tag{22}$$

In Eq. (22), j_1 is the total flux passing through the first component (InC) of conductivity Λ_1; j_{12} is the total flux passing successively through the second component (conductivity Λ_2) over the length $(L - l_2)$ and the first component over the length l_2; j'_{12} is the total flux passing successively through the second component over the length $(L - l_1)$ and the first component over the length l_1; j_2 is the total flux passing through the second component. The resistance R_i to the passage of flux j_i with a potential difference ΔU is by definition

$$j_i = \Delta U / R_i, \tag{23}$$

and the resistance R_i of the individual sections i is calculated from the simplest formula for plane walls

$$R_i = L_i/(\Lambda_i S_i), \tag{24}$$

where S_i, L_i are the cross-sectional area and length of section i. The equivalent coupling scheme of the resistances R_i in the elementary cell, according to Eq. (22), for the fluxes is shown in Fig. 3. The total resistance $R = L/(\Lambda S)$ is equated to the expression for R found from the scheme of Fig. 3

$$R^{-1} = R_1^{-1} + (R_2 + R_3)^{-1} + (R_4 + R_5)^{-1} + (R_6 + R_7)^{-1} + R_8^{-1}. \tag{25}$$

Substituting for the resistances R_i and geometric parameters of the elementary cell for various ranges of variation in the volume concentration m_1 from Tables 1 and 2 into Eq. (25), the final result obtained after appropriate manipulations is

TABLE 1. Resistance Values in Eq. (25)

Range of variation m_1	$R_1 a$ $(a=\Lambda_1 S/L)$	$(R_2+R_3)\,a$	$(R_4+R_5)\,a = (R_6+R_7)\,a$	$R_8 a$
$0 \leqslant m_1 \leqslant m_c$	∞	$\dfrac{1-(1-\nu)\,\bar{l}_2}{\nu \bar{S}_2}$	∞	$(\nu \bar{S}_3)^{-1}$
$m_c \leqslant m_1 \leqslant 0{,}5$	$(\bar{S}_1' H)^{-1}$	$\dfrac{1-(1-\nu)\,\bar{l}_2}{\nu \bar{S}_2}$	$\dfrac{1-(1-\nu)\,\bar{l}_1}{\nu \bar{l}_1 (1-\bar{l}_2)}$	$(\nu \bar{S}_3)^{-1}$
$0{,}5 \leqslant m_1 \leqslant (1-m_c)$	$(\bar{S}_1' H)^{-1}$	$\dfrac{\nu-(\nu-1)\,\bar{l}_2}{\bar{S}_2}$	$\dfrac{\nu-(\nu-1)\,\bar{l}_1}{\bar{l}_1 (1-\bar{l}_2)}$	$(\nu \bar{S}_3)^{-1}$
$(1-m_c) \leqslant m_1 \leqslant 1$	$(\bar{S}_1' H)^{-1}$	$\dfrac{\nu-(\nu-1)\,\bar{l}_2}{\bar{S}_2}$	∞	∞

TABLE 2. Geometric-Parameter Values in Eqs. (25) and (26)

Range of variation m_1	$\bar{S}_1'$	$\bar{S}_2$	$\bar{S}_3$	$\bar{l}_1$	$\bar{l}_2$
$0 \leqslant m_1 \leqslant m_c$	0	$m_1^{2/3}$	$1-m_1^{2/3}$	0	$m_1^{1/3}$
$m_c \leqslant m_1 \leqslant 0{,}5$	$\dfrac{1}{3}\,\dfrac{m_1-m_c}{1-m_c^{1/3}}$	$m_c^{2/3}-\bar{S}_1'$	$1-\bar{l}_2^2-2\bar{l}_1(1-\bar{l}_2)$	$(\bar{S}_1')^{1/2}$	$m_c^{1/3}$
$0{,}5 \leqslant m_1 \leqslant (1-m_c)$	$1-\bar{l}_2^2-2\bar{l}_1(1-\bar{l}_2)$	$m_c^{2/3}-\bar{S}_3$	$\dfrac{1}{3}\,\dfrac{m_2-m_c}{1-m_c^{1/3}}$	$(\bar{S}_3)^{1/2}$	$m_c^{1/3}$
$(1-m_c) \leqslant m_1 \leqslant 1$	$1-\bar{S}_2$	$m_2^{2/3}$	0	0	$m_2^{1/2}$

$$\Lambda = \Lambda_1 \bar{S}_1' H(\nu) + \Lambda_2 \nu^{\frac{\alpha-1}{2}} \left[\frac{\bar{S}_2}{1-(1-\nu^{\alpha})\,\bar{l}_2} + 2\,\frac{\bar{l}_1(1-\bar{l}_2)}{1-(1-\nu^{\alpha})\,\bar{l}_1} \right] + \Lambda_2 \bar{S}_3, \tag{26}$$

where $\alpha = 1$, if $m_j < 0.5$, and $\alpha = -1$, if $m_1 \geq 0.5$; the geometric parameters $\bar{l}_1$, $\bar{l}_2$, $\bar{S}_2$, $\bar{S}_3$, $\bar{S}_1'$ are determined in accordance with Table 2.

Turning briefly to the possible physical treatment of the correcting function H_1, H_1 may be determined, on the basis of Eqs. (3), (6), and (7), from the expression

$$H_1 = \frac{S_{\mathrm{InC}}}{S_1'}\,\frac{L}{L_{\mathrm{InC}}}. \tag{27}$$

When $m_c \leq m_1 \leq 0.5$ and $\nu = 0$, it is natural to assume that the mean cross-sectional area of the bridge in the model (Fig. 2b) is S_{InC}, i.e., $S_{\mathrm{InC}} = S_1'$, and so

$$H_1 = (L_{\mathrm{InC}}/L)^{-1}. \tag{28}$$

The ratio L_{InC}/L characterizes the branching of InC. In [5], in investigating the InC topology, it was noted that $L_{\mathrm{InC}}/L \sim (m_1 - m_c)^{-1}$ when $m_c \leq m_1 \leq 0.5$. In the present case, according to Eqs. (7) and (11)

$$\frac{L_{\mathrm{InC}}}{L} = \frac{1}{3}\,\frac{(1-m_c)^{1.6}}{1-m_c^{1/3}}\,\frac{1}{(m_1-m_c)^{0.6}}. \tag{29}$$

How the length of the infinite cluster changes with variation in the volume concentration is shown in Fig. 4: when $m_1 = m_2 = 0.5$, the InC length is equal to the length of the sample, i.e., $L_{\mathrm{InC}} = L$; when $m_1 \to m_c$, $L_{\mathrm{InC}} \to \infty$. Further, if it is assumed that the condition $L_{\mathrm{InC}} = L$ is satisfied when $m_1 > 0.5$, it follows from Eq. (28) that $H_1 = S_{\mathrm{InC}}/S_1'$, i.e., the function H_1 defined by Eqs. (7), (14), and (16) shows how the current line density in InC differs from the current line densities in the model.

In conclusion, discussion turns to the above assumption in Eq. (10), regarding the constancy of the central-core dimension l_2 of the infinite cluster with concentration variation $m_c \geq m_1 \geq 0.5$. In this case, transition to a model with interpenetrating components does not occur strictly at $m_1 = 0.5$ but in a certain range $m_1 = 0.5$-0.525 if m_c takes one of the values in the range 0.125-0.18. Thus, for example, the given transition occurs

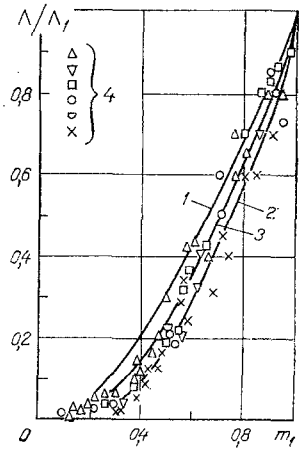

Fig. 5

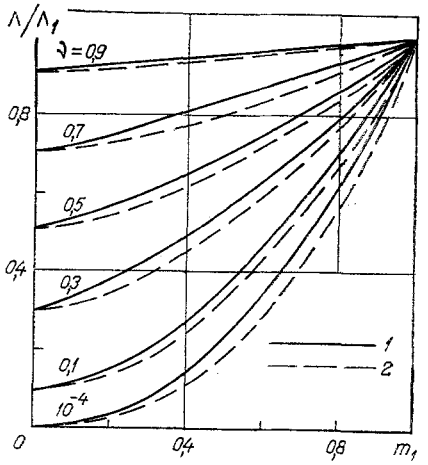

Fig. 6

Fig. 5. Curves of the generalized conductivity of an extremely inhomogeneous medium ($\nu = 0$) as a function of the volume concentration: according to numerical calculations of [1] (1) and [6] (2); according to Eq. (26) (3); and according to full-scale experiments for different materials [1-3].

Fig. 6. Curves of the generalized conductivity as a function of the concentration with various conductivity ratios of the inhomogeneous binary medium: 1) according to Eq. (26); 2) computer modeling [6].

at $m_1 = 0.507$ if $m_c = 0.15$ and at $m_1 = 0.525$ if $m_c = 0.18$. This model is plausible, since this structure is a statistical system, for which probabilistic laws are valid, and hence transition to a model with interpenetrating, geometrically equivalent components occurs in a certain range $m_1 = 0.5$-0.525. It would be possible to proceed by other means, and require that the following condition be satisfied: Transition to a model with geometrically equivalent interpenetrating components occurs at $m_1 = 0.5$. It is then necessary to assume that the volume concentration of the isolated cluster m_c is not constant, but varies. Calculations show that the final values of the conductivity Λ obtained in realizing these models differ insignificantly. In the present work, the first, simpler model is realized; other possible variants of this basic model will be considered in the future.

The dependence $\Lambda/\Lambda_1 = f(m_1)$ when $\nu = 0$, plotted from the results of numerical calculations and full-scale experiments, is shown in Fig. 5. As follows from Fig. 5, the dependence occupies a definite region, which is a consequence not only of the accuracy of the experiments and computer calculations but also of the statistical character of the process under study. Curves of $\Lambda/\Lambda_1 = f(m_1)$ plotted from numerical-modeling data [6] and from Eq. (26) lie in the central part of the region. It may be that the two curves are equally likely, although the difference between them sometimes reaches 20%, whereas the boundaries of the region are up to 50% apart.

Curves of $\Lambda/\Lambda_1 = f(m_1, \nu)$ plotted according to the results of numerical modeling [6] and Eq. (26) are compared in Fig. 6. Taking account of the above remarks, it may be concluded that there is good agreement between the results of the two different methods of analyzing transfer processes through inhomogeneous media.

NOTATION

Λ, effective conductivity; Λ_i, conductivity of the i-th component; m_i, volume concentration of the i-th component; m_c, flow threshold; H_1, correcting function; $\nu = \Lambda_2/\Lambda_1$; L_{InC}, S_{InC}, length and cross-sectional area of the infinite cluster.

LITERATURE CITED

1. S. Kirkpatrik, "Percolation and conductivity," in: Theory and Properties of Disordered Materials [Russian translation], V. L. Bonch-Bruevich (ed.), Mir, Moscow (1977).
2. B. I. Shklovskii and A. L. Éfros, "Flow theory and the conductivity of strongly inhomogeneous media," Usp. Fiz. Nauk, 117, No. 3, 401-435 (1975).
3. G. N. Dul'nev and V. V. Novikov, "Conductivity of inhomogeneous systems," Inzh.-Fiz. Zh., 36, No. 5, 901-910 (1979).

4. G. N. Dul'nev and Yu. P. Zarichnyak, Thermal Conductivity of Mixtures and Composite Materials [in Russian], Énergiya, Leningrad (1974).
5. A. S. Skal and B. I. Shklovskii, "Topology of an infinite cluster in flow theory and theory of jump conductivity," Fiz. Tekh. Poluprovodn., 8, 1586-1591 (1974).
6. G. N. Dul'nev, V. K. Kruglikov, and E. V. Sakhova, "Mathematical modeling of heterogeneous isotropic systems," Inzh.-Fiz. Zh., 41, No. 5, 859-864 (1981).

KINETICS OF THE SELF-SIMILAR CONDITIONS OF DIFFUSIONAL COMBUSTION OF POLYDISPERSE LIQUID FUEL

Yu. M. Goldobin

UDC 621.1.016:536.46

On the basis of the kinetic equation for the particle distribution function with respect to the radius, a method is proposed for calculating the processes of heat and mass transfer in the combustion of liquid fuel of a polydisperse melt.

In the ignition of a liquid-drop or dustlike fuel in various devices, the determining role is played by the kinetics of particle combustion in a medium of oxidant diluted with inert gas. The problems arising here are very complex in view of their nonlinearity, and therefore they are solved, especially for the case of various engineering devices, by means of experimental methods or numerical calculations using a computer. In the latter case, the calculation of the combustion kinetics of a polydisperse particle system is undertaken by dividing the initial particle size distribution into narrow fractions [1-3]. An alternative approach which has been successfully used in calculations of vaporization and solution [4-6] is to use the kinetic equation for the particle distribution with respect to the radius.

In the present work, on the basis of this approach [5, 6], the simplest model of quasidiffusional combustion of polydisperse liquid-fuel drops in a volume with adiabatic walls is considered.

As in [6], it is assumed that the fuel drops are spherical in form; that their mass concentration is small; that the product consumption corresponds to stoichiometry of the reaction; and that there is no breakdown or coagulation of the drops; that the fuel is injected into gas containing oxidant with a temperature of the medium $T_{me.0}$ above the ignition temperature. The assumptions adopted also hold in real conditions.

Polydispersity of the drops in the combustion process will be taken into account by the kinetic equation for the particle distribution function with respect to the radius $f(r_S, t)$ [5, 6]

$$\frac{\partial f(r_S, t)}{\partial t} + \frac{\partial}{\partial r_S}[f(r_S, t) W(r_S, t)] = 0, \quad (1)$$

for which the following relation will hold

$$dN = N_0 f(r_S, t)\, dr_S; \quad f(r_S, 0) = f_0(r_S); \quad \int_0^\infty f_0(r_S)\, dr_S = 1. \quad (2)$$

In [6], the distribution function $f(r_S, t)$ for self-similar quasisteady conditions of evaporation was obtained in general form. Analogous determination of $f(r_S, t)$ is possible in combustion if the combustion rate of a single fuel drop $W(r_S, t)$ is known.

The function $W(r_S, t)$ is obtained on the basis of the quasisteady diffusional combustion of a drop, under the assumption that the simplest reaction between fuel vapor and oxidant occurs in the flame. At high temper-

S. M. Kirov Ural Polytechnic Institute, Sverdlovsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 45, No. 3, pp. 452-457, September, 1983. Original article submitted June 1, 1982.

0022-0841/83/4503-1037$07.50